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STUDY PACKAGE

Subject : PHYSICS

Topic : ERROR IN MEASUREMENTS & INSTRUMENS

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ERRORS

Whenever an experiment is performed, two kinds of errors can appear in the measured quantity.

(1) indeterminate and (2) determinate (or systematic) errors.

1. Indeterminate errors appear randomly because of operator, fluctuations in external conditions and variability of measuring instruments. The effect of indeterminate error can be some what reduced by taking the average of measured values. Indeterminate errors have no fixed sign or size.
2. Determinate errors occur due to error in the procedure, or miscalibration of the intrument etc. Sucherrors have same size and sign for all the measurements. Such errors can be determined.

A measurement with relatively small indeterminate error is said to have high precision. A measurement with small indeterminate error and small determinate error is said to have high accuracy.

The experimental error [uncertainty] can be expressed in several standard ways.

Error limits $Q \pm \Delta Q$ is the measured quantity and ΔQ is the magnitude of its limit of error. This expresses the experimenter's judgment that the 'true' value of Q lies between $Q - \Delta Q$ and $Q + \Delta Q$. This entire interval within which the measurement lies is called the range of error. Indeterminate errors are expressed in this form.

Absolute Error

Error may be expressed as absolute measures, giving the size of the error in a quantity in the same units as the quantity itself.

Relative (or Fractional) Error

Error may be expressed as relative measures, giving the ratio of the quantity's error to the quantity itself. In general

$$\text{relative error} = \frac{\textit{absolute error in a measurement}}{\textit{size of the measurement}}$$

We should know the error in the measurement because these errors propagate through the calculations to produce errors in results.

A. *Determinate errors* : They have a known sign.

1. Suppose that a result R is calculated from the sum of two measured quantities A and B. We'll use a and b to represent the error in A and B respectively. r is the error in the result R. Then

$$(R + r) = (A + B) + (a + b)$$

The error in R is therefore : $r = a + b$.

Similarly, when two quantities are subtracted, the determinate errors also get subtracted.

2. Suppose that a result R is calculated by multiplying two measured quantities A and B. Then $R = AB$.

$(R + r) = (A + a)(B + b) = AB + aB + Ab + ab$

$\Rightarrow \quad \frac{r}{R} = \frac{aB + bA}{AB} = \frac{a}{A} + \frac{b}{B}$. Thus when two quantities are multiplied, their relative determinate error add.

3. **Quotient rule :** When two quantities are divided, the relative determinate error of the quotient is the relative determinate error of the numerator minus the relative determinate error of the denominator. Thus if $R = \frac{A}{B}$ then

$\frac{r}{R} = \frac{a}{A} - \frac{b}{B}$

4. **Power rule :** When a quantity Q is raised to a power, P, the relative determinate error in the result is P times the relative determinate error in Q.

If $R = Q^P$, $\quad \frac{r}{R} = P \times \frac{q}{Q}$

This also holds for negative powers,

5. The quotient rule is not applicable if the numerator and denominator are dependent on each other.

e.g if $R = \frac{XY}{X+Y}$. We cannot apply quotient rule to find the error in R. Instead we write the equation as follows

$\frac{1}{R} = \frac{1}{X} + \frac{1}{Y}$. Differentiating both the sides, we get

$$-\frac{dR}{R^2} = -\frac{dX}{X^2} - \frac{dY}{Y^2}. \qquad \text{Thus} \quad \frac{r}{R^2} = \frac{x}{X^2} + \frac{y}{Y^2} \quad \text{or} \quad \frac{r}{R} = \frac{R}{X}\times\frac{x}{X} + \frac{R}{Y}\times\frac{y}{Y}$$

B. ***Indeterminate error* :** They have unknown sign. Thus they are represented in the form A ± a.

Here we are only concerned with *limits of error*. We must assume a "worst-case" combination. In the case of substraction, A – B, the worst-case deviation of the answer occurs when the errors are either + a and – b or – a and + b. In either case, the maximum error will be (a + b)

1. ***Addition and subtraction rule* :** The absolute indeterminate errors add.
Thus if R = A + B, r = a + b
and if R = A – B, r = a + b

2. ***Product and quotient rule* :** The relative indeterminate errors add.

Thus if R = AB, $\frac{r}{R} = \frac{a}{A} + \frac{b}{B}$

and if $R = \frac{A}{B}$, then also $\frac{r}{R} = \frac{a}{A} + \frac{b}{B}$

3. ***Power rule* :** When a quantity Q is raised to a power P, the relative error in the result is P times the relative error in Q. This also holds for negative powers.

If $R = Q^P$, $\frac{r}{R} = P \times \frac{q}{Q}$

Examples

1. A student finds the constant acceleration of a slowly moving object with a stopwatch. The equation used is $S = (1/2)AT^2$. The time is measured with a stopwatch, the distance, S with a meter stick. What is the acceleration and its estimated error?
S = 2 ± 0.005 meter.
T = 4.2 ± 0.2 second.

Sol: We use capital letters for quantities, lower case for errors. Solve the equation for the result, a. $A = 2S/T^2$. Its indeterminate-error equation is $\frac{a}{A} = 2\frac{t}{T} + \frac{s}{S}$

Thus $A = 0.23 \pm 0.02$ m/s^2.

SIGNIFICANT DIGITS

Significant figures are digits that are statistically significant. There are two kinds of values in science :

1. Measured Values
2. Computed Values

The way that we identify the proper number of significant figures in science are different for these two types.

MEASURED VALUES

Identifying a measured value with the correct number of significant digits requires that the instrument's calibration be taken into consideration. The last significant digit in a measured value will be the first estimated position.For example, a metric ruler is calibrated with numbered calibrations equal to 1 cm. In addition, there will be ten unnumbered calibration marks between each numbered position. (each equal to 0.1 cm). Then one could with a little practice estimate between each of those marking. (each equal to 0.05 cm). That first estimated position would be the last significant digit reported in the measured value. Let's say that we were measuring the length of a tube, and it extended past the fourteenth numbered calibration half way between the third and fourth unnumbered mark. The metric ruler was a meter stick with 100 numbered calibrations. The reported measured length would be 14.35 cm. Here the total number of significant digits will be 4.

COMPUTED VALUE

The other type of value is a computed value. The proper number of significant figures that a computed value should have is decided by a set of conventional rules. However before we get to those rules for computed values we have to consider how to determine how many significant digits are indicated in the numbers being used in the math computation.

A. Rules for determining the number of significant digits in number with indicated decimals.

1. All non-zero digits (1-9) are to be counted as significant.
2. Zeros that have any non-zero digits anywhere to the LEFT of them are considered significant zeros.
3. All other zeros not covered in rule (ii) above are NOT be considered significant digits.

For example : 0.0040000

The 4 is obviously to be counted significant (Rule-1), but what about the zeros? The first three zeros would not be considered significant since they have no non-zero digits anywhere to their left (Rule-3). The last four zeros would all be considered significant since each of them has the non-zero digit 4 to their left (Rule-2). Therefore the number has a total of five significant digits.

Here is another example : 120.00420

The digit 1, 2, 4 and 2 are all considered significant (Rule-1). All zeros are considered significant since they have non-zero digits somewhere to their left (Rule-2). So there are a total of eight significant digits.

B. Determining the number of significant digits if number is not having an indicated decimal.

The decimal indicated in a number tells us to what position of estimation the number has been indicated. But what about 1,000,000?

Notice that there is no decimal indicated in the number. In other words, there is an ambiguity concerning the estimated position. This ambiguity can only be clarified by placing the number in exponential notation.

For example : If I write the number above in this manner.

$$1.00 \times 10^6$$

I have indicated that the number has been recorded with three significant digits. On the other hand, if I write the same number as : 1.0000×10^6

I have identified the number to have 5 significant digits. Once the number has been expressed in exponential notation form then the digits that appear before the power of ten will all be considered significant. So for example : 2.0040×10^4 will have five significant digits. This means that unit conversion will not change the number of significant digits. Thus 0.000010 km = 1.0 cm = 0.010 m = 1.0×10^{-2} m = 1.0×10^{-5} km

Rule for expressing proper number of significant digits in an answer from multiplication or division

For multiplication AND division there is the following rule for expressing a computed product or quotient with the proper number of significant digits.

The product or quotient will be reported as having as many significant digits as the number involved in the operation with the least number of significant digits.

For example : $0.000170 \times 100.40 = 0.017068$

The product could be expressed with no more that three significant digits since 0.000170 has only three significant digits, and 100.40 has five. So according to the rule the product answer could only be expressed with three significant digits. Thus the answer should be 0.0171 (after rounding off)

Another example : $2.000 \times 10^4 / 6.0 \times 10^{-3} = 0.33 \times 10^7$

The answer could be expressed with no more that two significant digits since the least digited number involved in the operation has two significant digits.

Sometimes this would required expressing the answer in exponential notation.

For example : $3.0 \times 800.0 = 2.4 \times 10^3$

The number 3.0 has two significant digits and then number 800.0 has four. The rule states that the answer can have no more than two digits expressed. However the answer as we can all see would be 2400. How do we express the answer 2400 while obeying the rules? The only way is to express the answer in exponential notation so 2400 could be expressed as : 2.4×10^3

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Rule for expressing the correct number of significant digits in an addition or substraction :

The rule for expressing a sum or difference is considerably different than the one for multiplication of division. The sum or difference can be no more precise than the least precise number involved in the mathematical operation. Precision has to do with the number of positions to the RIGHT of the decimal. The more position to the right of the decimal, the more precise the number. So a sum or difference can have no more indicated positions to the right of the decimal as the number involved in the operation with the LEAST indicated positions to the right of its decimal.

For example : 160.45 + 6.732 = 167.18 (after rounding off)
The answer could be expressed only to two positions to the right of the decimal, since 160.45 is the least precise.
Another example : 45.621 + 4.3 – 6.41 = 43.5 (after rounding off)
The answer could be expressed only to one position to the right of the decimal, since the number 4.3 is the least precise number (i.e. having only one position to the right of its decimal). Notice we aren't really determining the total number of significant digits in the answer with this rule.

Rules for rounding off digits :

There are a set of conventional rules for rounding off.

1. Determine according to the rule what the last reported digit should be.
2. Consider the digit to the right of the last reported digit.
3. If the digit to the right of the last reported digit is less than 5 round it and all digits to its right off.
4. If the digit to the right of the last reported digit is greater than 5 round it and all digits to its right off and increased the last reported digit by one.
5. If the digit to the right of the last reported digit is a 5 followed by either no other digits or all zeros, round it and all digits to its right off and if the last reported digit is odd round up to the next even digit. If the last reported digit is even then leave it as is.

For example if we wish to round off the following number to 3 significant digits : 18.3682
The last reported digits would be the 3. The digit to its right is a 6 which is greater than 5. According to the Rule-4 above, the digit 3 is increased by one and the answer is : 18.4

Another example : Round off 4.565 to three significant digits.
The last reported digit would be the 6. The digit to the right is a 5 followed by nothing. Therefore according to Rule-5 above since the 6 is even it remains so and the answer would be 4.56.

EXPERIMENTS

(i) Measurement of length

The simplest method measuring the length of a straight line is by means of a meter scale. But there exists some limitation in the accuracy of the result:
(i) the dividing lines have a finite thickness.
(ii) naked eye cannot correctly estimate less than 0.5 mm
For greater accuracy devices like
(a) Vernier callipers (b) micrometer scales (screw gauge) are used.

VERNIER CALLIPERS:

It consists of a main scale graduated in cm/mm over which an auxiliary scale (or Vernier scale) can slide along the length. The division of the Vernier scale being either slightly longer and shorter than the divisions of the main scale.

Least count of Vernier Callipers

The least count or Vernier constant (v. c) is the minimum value of correct estimation of length without eye estimation. If N division of vernier coincides with (N-1) division of main scale, then

Vernier constant = 1 ms – 1 vs = $\left(1-\frac{N-1}{N}\right)$ms = $\frac{1\text{ms}}{N}$, which is equal to the value of the smallest division on the main scale divided by total number of divisions on the vernier scale.

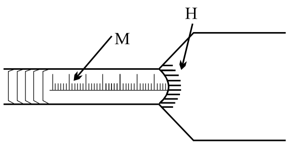
Zero error:

If the zero marking of main scale and vernier callipers do not coincide, necessary correction has to be made for this error which is known as zero error of the instrument.

If the zero of the vernier scale is to the right of the zero of the main scale the zero error is said to be positive and the correction will be negative and vice versa.

SCREW GAUGE (OR MICROMETER SCREW)

In general vernier callipers can measure accurately upto 0.01 em and for greater accuracy micrometer screw devices e.g. screw gauge, spherometer are used. These consist of accurately cut screw which can be moved in a closely fitting fixed nut by tuming it axially. The instrument is provided with two scales:



(i) The main scale or pitch scale M graduated along the axis of the screw.

(ii) The cap-scale or head scale H round the edge of the screw head.

Constants of the Screw Gauge

(a) **Pitch :** The translational motion of the screw is directly proportional to the total rotation of the head. The pitch of the instrument is the distance between two consecutive threads of the screw which is equal to the distance moved by the screw due to one complete rotation of the cap. Thus for 10 rotation of cap =5 mm, then pitch = 0.5 mm

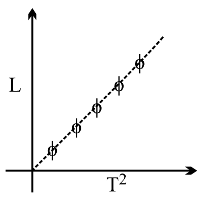
(b) **Least count :** In this case also, the minimum (or least) measurement (or count) of length is equal to one division on the head scale which is equal to pitch divided by the total cap divisions. Thus in the aforesaid Illustration:, if the total cap division is 100, then least count = 0.5mm/100 = 0.005 mm

Zero Error : In a perfect instrument the zero of the heat scale coincides with the line of graduation along the screw axis with no zero-error, otherwise the instrument is said to have zero-error which is equal to the cap reading with the gap closed. This error is positive when zero line or reference line of the cap lies above the line of graduation and versa. The corresponding corrections will be just opposite.

(ii) **Measurement of g using a simple pendulum**

A small spherical bob is attached to a cotton thread and the combination is suspended from a point A. The length of the thread (L) is read off on a meter scale. A correction is added to L to include the finite size of the bob and the hook. The corrected value of L is used for further calculation. The bob is displaced slightly to one side and is allowed to oscillate, and the total time taken for 50 complete oscillations is noted on a stop-watch. The time period (T) of a single oscillation is now calculated by division.

Observations are now taken by using different lengths for the cotton thread (L) and pairs of values of L and T are taken. A plot of L v/s T^2, on a graph, is linear. g is given by $g = 4\pi^2 \frac{L}{T^2}$



The major errors in this experiment are

(a) **Systematic :** Error due to finite amplitude of the pendulum (as the motion is not exactly SHM). This may be corrected for by using the correct numerical estimate for the time period. However the practice is to ensure that the amplitude is small.

(b) **Statistical :** Errors arising from measurement of length and time.

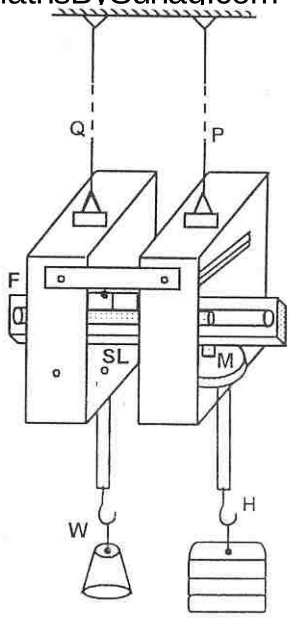
$$\frac{\delta g}{g} = \frac{\delta L}{L} + 2\left(\frac{\delta T}{T}\right)$$

The contributions to δL, δT are both statistical and systematic. These are reduced by the process of averaging. The systematic error in L can be reduced by plotting several values of L vs T^2 and fitting to a straight line. The slope of this fit gives the correct value of L/T^2

(iii) Determination of Young's Modulus by Searle's Method

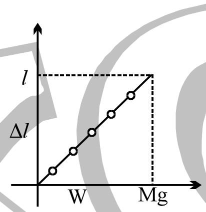
The experimental set up consists of two identical wires P and Q of uniform cross section suspended from a fixed rigid support. The free ends of these parallel wires are connected to a frame F as shown in the figure. The length of the wire Q remains fixed while the load L attached to the wire P through the frame F is varied in equal steps so as to produce extension along .the length. The extension thus produced is measured with the help of spirit level SL and micrometer screw M attached to the F frame on the side of the experimental wire.On placing the slotted weights on the hanger H upto a permissible value (half of the breaking force) the wire gets extended by small amount and the spirit level gets disturbed from horizontal setting. This increase in length is measured by turning the micrometer screw M upwards so as to restore the balance of the spirit level. If n be the number of turns of the micrometer screw and f be the difference in the cap reading, the increase in length *M* is obtained by

$$\Delta l = n \times \text{pitch} + f \times \text{least count}$$



The load on the hanger is reduced in the same steps and spirit level is restored to horizontal position. The mean of these two observations gives the true increase in length of the wire corresponding to the given value of load. From the data obtained, a graph showing extension (Δl) against the load (W) is plotted which is obtained as a straight line passing through the origin. The slope of the line gives

$$\tan\theta = \frac{l}{W} = \frac{l}{Mg}$$



Now, stress $= \dfrac{Mg}{\pi r^2}$ and strain $= \dfrac{l}{L}$

$$Y = \text{Stress/ strain} = \frac{MgL}{\pi r^2 l} = \frac{L}{\pi r^2 \tan\theta}$$

With known values of initial length L, radius r of the experimental wire and tanθ, Young's modulus Y can be calculated.

(iv) Specific Heat of a liquid using a calorimeter:

The principle is to take a known quantity of liquid in an insulated calorimeter and heat it by passing a known current (i) through a heating coil immersed within the liquid for a known length of time (t). The mass of the calorimeter (m_1) and, the combined mass of the calorimeter and the liquid (m_2) are measured. The potential drop across the heating coil is V and the maximum temperature of the liquid is measured to θ_2.

The specific heat of the liquid (S_l) is found by using the relation

$$(m_2 - m_1)S_l(\theta_2 - \theta_0) + m_1S_c(\theta_2 - \theta_0) = i.\ V.\ t$$

or, $(m_2 - m_1)S_l + m_1S_c = i.\ V.\ t\,/(\theta_2 - \theta_0)$ ..... (1)

Here, θ_0 is the room temperature, while S_c is the specific heat of the material of the calorimeter and the stirrer. If S_c is known, then S_l can be determined.

On the other hand, if S_c is unknown: one can either repeat the experiment with water or a different mass of the liquid and use the two equations to eliminate m_1S_c.

The sources of error in this experiment are errors due to improper connection of the heating coil, radiation, apart from statistical errors in measurement.

The direction of the current is reversed midway during the experiment to remove the effect of any differential contacts, radiation correction is introduced to take care of the second major source of systematic error.

Radiation correction: The temperature of the system is recorded for half the length of time t, i.e. t/2, where t is the time during which the current was switched on} after the current is switched off. The fall in temperature δ, during this interval is now added to the final temperature θ_2 to give the corrected final temperature: $\theta'_2 = \theta_2 + \delta$

This temperature is used in the calculation of the specific heat, S_l.

Error analysis :

After correcting for systematic errors, equation (i) is used to estimate the remaining errors.

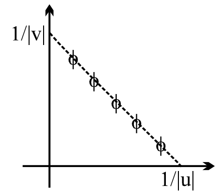
(v) Focal length of a concave mirror and a convex lens using the u-v method.

In this method one uses an optical bench and the convex lens (or the concave mirror) is placed on the holder. The position of the lens is noted by reading the scale at the bottom of the holder. A bright object (a filament lamp or some similar object) is placed at a fixed distance (u) in front of the lens (mirror).

The position of the image (v) is determined by moving a white screen behind the lens until a sharp image is obtained (for real images).

For the concave mirror, the position of the image is determined by placing a sharp object (a pin) on the optical bench such that the parallax between the object pin and the image is nil.

A plot of |u| versus |v| gives a rectangular hyperbola. A plot of $\frac{1}{|v|}$ vs $\frac{1}{|u|}$ gives a straight line.



The intercepts are equal to $\frac{1}{|f|}$, where f is the focal length.

Error : The systematic error in this experiment is mostly due to improper position of the object on the holder. This error maybe eliminated by reversing the holder (rotating the holder by 180° about the vertical) and then taking the readings again. Averages are then taken.

The equation for errors gives:

$$\left|\frac{\delta f}{f}\right| = \left|\frac{\delta u}{u}\right| + \left|\frac{\delta v}{v}\right| + \frac{|\delta u| + |\delta v|}{|u| + |v|}$$

The errors δu, δv correspond to the error in the measurement of u and v.

Index Error or Bench Error and its correction: In an experiment using an optical bench we are required to measure the object and image distances from the pole or vertex on the mirror. The distance between the tip of the needles and the pole of the mirror is the actual distance. But we practically measure distances between the indices with the help of the scale engraved on the bench. These distances are called the observed distances. The actual distances may not be equal to the observed distances and due to this reason an error creeps in the measurement of the distances. This error is called the index or the bench error.

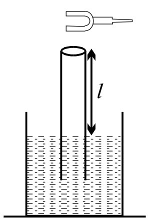
Index Error = Observed distance – actual distance and

Index Correction = Actual – observed distance

Note: Index correction whether positive or negative, is always added algebraically to the observed distance to get the corrected distance.

(vi) Speed of sound using resonance column

A tuning fork of known frequency (f) is held at the mouth of a long tube, which is dipped into water as shown in the figure. The length (l_1) of the air column in the tube is adjusted until it resonates with the tuning fork. The air temperature and humidity are noted. The length of the tube is adjusted again until a second resonance length (l_2) is found (provided the tube is long)



Then, $l_2 - l_1 = \lambda / 2$, provided l_1, l_2 are resonance lengths for adjacent resonances.

∴ $\lambda = 2(l_2 - l_1)$, is the wavelength of sound.

Since the frequency f, is known; the velocity of sound in air at the temperature (θ) and humidity (h) is given by $C = f\lambda = 2(l_2 - l_1)f$

It is also possible to use a single measurement of the resonant length directly, but, then it has to be corrected *for* the "end effect":

$\lambda(\text{fundamental}) = 4(l_1 + 0.3d)$, where d = diameter

Errors : The major systematic errors introduced are due to end effects in (end correction) and also due to excessive humidity.

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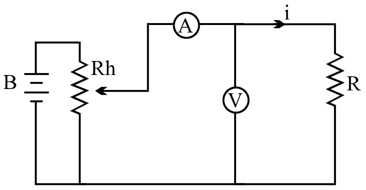
Random errors are given by

$$\frac{\delta C}{C} = \frac{\delta(l_2 - l_1)}{l_2 - l_1} = \frac{\delta l_2 + \delta l_1}{l_2 - l_1}$$

(vii) Verification of Ohm's law using voltmeter and ammeter

A voltmeter (V) and an ammeter (A) are connected in a circuit along with a resistance R as shown in the figure, along with a battery B and a rheostat, Rh Simultaneous readings of the current i and the potential drop V are taken by changing the resistance in the rheostat (Rh). A graph of V vs i is plotted and it is found to be linear (within errors).



The magnitude of R is determined by either

(a) taking the ratio $\frac{V}{i}$ and then

(b) fitting to a straight line: V = iR, and determining the slope R.

Errors :

Systematic errors in this experiment arise from the current flowing through V (finite resistance of the voltmeter), the Joule heating effect in the circuit and the resistance of the connecting wires/ connections of the resistance. The effect of Joule heating may be minimsed by switching on the circuit for a short while only, while the effect of finite resistance of the voltmeter can be overcome by using a high resistance instrument or a potentiometer. The lengths of connecting wires should be minimised as much as possible.

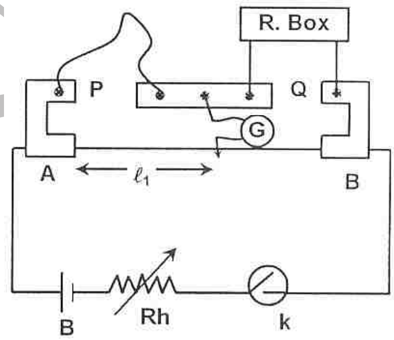
Error analysis :

The error in computing the ratio $R = \frac{V}{i}$ is given by $\left|\frac{\delta R}{R}\right| = \left|\frac{\delta V}{V}\right| + \left|\frac{\delta i}{i}\right|$

where δV and δi are of the order of the least counts of the instruments used.

(viii) Specific resistance of the material of a wire using a meter bridge :

A known length (l) of a wire is connected in one of the gaps (P) of a metre bridge, while a Resistance Box is inserted into the other gap (Q). The circuit is completed by using a battery (B), a Rheostat (Rh), a Key (K) and a galvanometer (G). The balance length (l) is found by closing key k and momentarily connecting the galvanometer until it gives zero deflection (null point). Then,



$$\frac{P}{Q} = \frac{l}{100 - l} \qquad ....(1)$$

using the expression for the meter bridge at balance. Here, represents the resistance of the wire while Q represents the resistance in the resistance box. The key K is open when the circuit is not in use.

The resistance of the wire, $P = \rho\frac{L}{\pi r^2} \Rightarrow \rho = \frac{\pi r^2}{L}P \qquad ....(2)$

where r is the radius of wire and L is the length of the wire, r is measured using a screw gauge while L is measured with a scale.

Errors : The major systematic errors in this experiment are due to the heating effect, end corrections introduced due to shift of the zero of the scale at A and B, and stray resistances in P and Q, and errors due to non-uniformity of the meter bridge wire.

Error analysis : End corrections can be estimated by including known resistances P_1 and Q_1 in the two ends and finding the null point:

$$\frac{P_1}{Q_1} = \frac{l_1 + \alpha}{100 - l_1 + \beta}$$ ..... (2), where α and β are the end corrections.

When the resistance Q_1 is placed in the left gap and P_1 in the right gap,

$$\frac{Q_1}{P_1} = \frac{l_2 + \alpha}{100 - l_2 + \beta} \quad ..... (3)$$

which give two linear equation for finding α and β.

In order that α and β be measured accurately, P_1 and Q_1 should be as different from each other as possible. For the actual balance point,

$$\frac{P}{Q} = \frac{l + \alpha}{100 - l + \beta} = \frac{l_1'}{l_2'},$$

Errors due to non-uniformity of the meter bridge wire can be minimised by interchanging the resistances in the gaps P and Q.

$$\therefore \quad \frac{\delta P}{P} = \left|\frac{\delta l_1'}{l_1'}\right| + \left|\frac{\delta l_2'}{l_2'}\right|$$

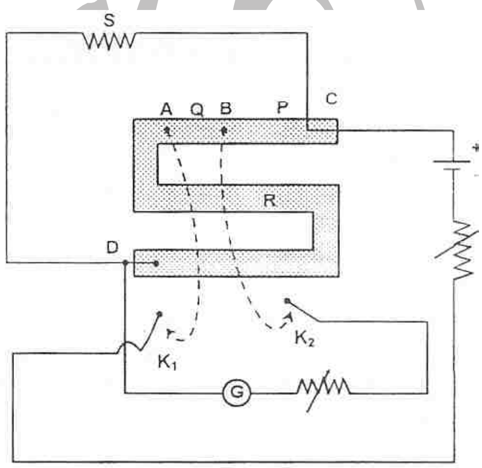
where, $\delta l'_1$ and $\delta l'_2$ are of the order of the least count of the scale.

The error is, therefore, minimum if $l'_1 = l'_2$ i.e. when the balance point is in the middle of the bridge. The error in ρ is

$$\frac{\delta P}{P} = \frac{2\delta r}{r} + \frac{\delta L}{L} + \frac{\delta P}{P}$$

(ix) Measurement of unknown resistance using a P.O. Box

A P.O. Box can also be used to measure an unknown resistance. It is a Wheatstone Bridge with three arms P, Q and R; while the fourth arm(s) is the unknown resistance. P and Q are known as the ratio arms while R is known at the rheostat arm.



At balance, the unknown resistance

$$S = \left(\frac{P}{Q}\right) R \quad .....(1)$$

The ratio arms are first adjusted so that they carry 100 Ω each. The resistance in the rheostat arm is now adjusted so that the galvanometer deflection is in one direction, if $R = R_0$ (Ohm) and in the opposite direction when $R = R_0 + 1$ (ohm).

This implies that the unknown resistance, S lies between R_0 and $R_0 + 1$ (ohm). Now, the resistance in P and Q are made 100 Ω and 1000 Ω respectively, and the process is repeated.

Equation (1) is used to compute S.

The ratio P/Q is progressively made 1 : 10, and then 1 : 100. The resistance S can be accurately measured.

Errors : The major sources of error are the connecting wires, unclear resistance plugs, change in resistance due to Joule heating, and the insensitivity of the Wheatstone bridge.

These may be removed by using thick connecting wires, clean plugs, keeping the circuit on for very brief periods (to avoid Joule heating) and calculating the sensitivity.

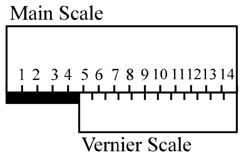
In order that the sensitivity is maximum, the resistance in the arm P is close to the value of the resistance S.

EXERCISE

Q.1 In a Vernier Calipers (VC), N divisions of the main scale coincide with N + m divisions of the vernier scale. What is the value of m for which the instrument has minimum least count?

(A) 1 (B) N (C) Infinity (D) N/2

Q.2 Consider the vernier calipers as shown, the instrument has no zero error. What is the length of the rod shown, if 1 msd = 1mm? Use 7 msd = 8 vsd.

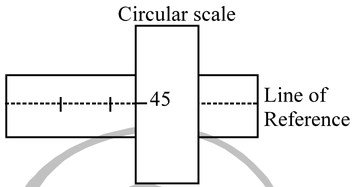


(A) 4.6 mm (B) 4.5 mm (C) 4.3 mm (D) none

Q.3 In a vernier calipers the main scale and the vernier scale are made up different materials. When the room temperature increases by $\Delta T°C$, it is found the reading of the instrument remains the same. Earlier it was observed that the front edge of the wooden rod placed for measurement crossed the N^{th} main scale division and N + 2 msd coincided with the 2^{nd} vsd. Initially, 10 vsd coincided with 9 msd. If coefficient of linear expansion of the main scale is α_1 and that of the vernier scale is α_2 then what is the value of α_1 / α_2? (Ignore the expansion of the rod on heating)

(A) 1.8 / (N) (B) 1.8 /(N+2) (C) 1.8 / (N–2) (D) None

Q.4 Consider a screw gauge without any zero error. What will be the final reading corresponding to the final state as shown? It is given that the circular head translates P msd in N rotations. One msd is equal to 1mm.

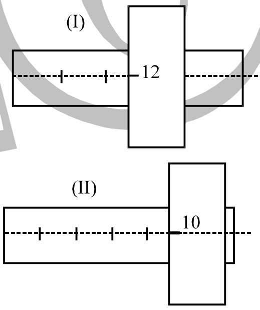


The circular scale has 100 divisions

(A) (P/N) (2 + 45/100) mm (B) (N/P) (2+45/N)mm

(C) P (2/N + 45/100)mm (D) $\left(2+\frac{45}{100}\times\frac{P}{N}\right)$mm

Q.5 A screw gauge has some zero error but its value is unknown. We have two identical rods. When the first rod is inserted in the screw, the state of the instrument is shown by diagram (I). When both the rods are inserted together in series then the state is shown by the diagram (II). What is the zero error of the instrument? 1 msd = 100 csd = 1mm



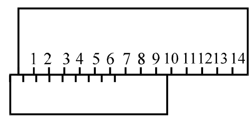
(A) –0.16 mm (B) +0.16 mm

(C) +0.14 mm (D) –0.14 mm

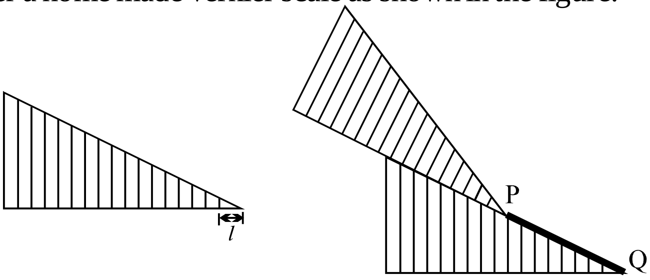
Q.6 The VC shown in the diagram has zero error in it (as you can see). It is given that 9 msd = 10 vsd.

(i) What is the magnitude of the zero error?

(ii) The observed reading of the length of a rod measured by this VC comes out to be 5.4 mm. If the vernier had been error free then ___msd would have coincided with ___ vsd.



Q.7 Consider a home made vernier scale as shown in the figure.



In this diagram, we are interested in measuring the length of the line PQ. If the angle of both the inclines is equal to θ then what is the least count of the instrument.

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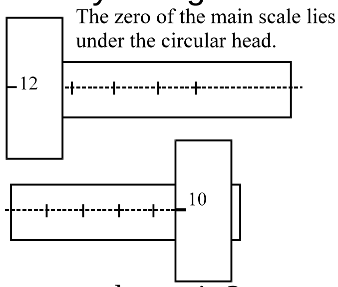
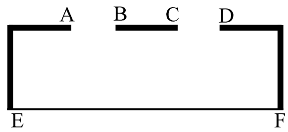
Q.8 The diagram shows the initial and the final state of SG, which has zero error in it. What can be the length of the object? 1 msd = 100 csd

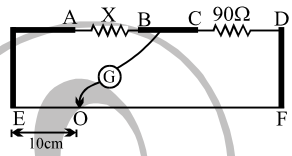


Q.9 In a meter bridge set up, which of the following should be the properties of the one meter long wire?
(A) High resistivity and low temperature coefficient
(B) Low resistivity and low temperature coefficient
(C) Low resistivity and high temperature coefficient
(D) High resistivity and high temperature coefficient

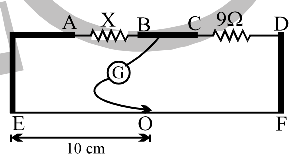
Q.10 Make the appropriate connections in the meter bridge set up shown. Resistance box is connected between _____. Unknown resistance is connected between______. Battery is connected between.
Options:
(A) AB (B) CD (C) EF (D) None



Q.11 Let the end error on the LHS and RHS be equal to one cm. For the balance point at O, find out the % tage error in the value of X? (If the end error is 1 cm from both sides then it means the corrected reading will become 10cm + 1cm from LHS and 90cm + 1cm from the RHS)
(A) 4.2% (B) 8.1% (C) 9.2% (D) None

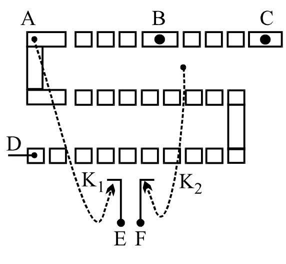


Q.12 Consider the MB shown in the diagram, let the resistance X have temperature coefficient α_1 and the resistance from the RB have the temperature coefficient α_2. Let the reading of the meter scale be 10cm from the LHS. If the temperature of the two resistance increase by small temperature ΔT then what is the shift in the position of the null point? Neglect all the other changes in the bridge due to temperature rise.

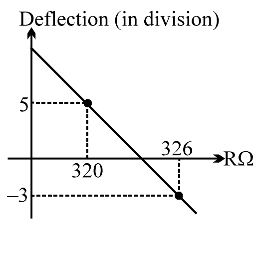


(A) $9(\alpha_1 - \alpha_2)\Delta T$ (B) $9(\alpha_1 + \alpha_2)\Delta T$ (C) $\frac{1}{9}(\alpha_1 + \alpha_2)\Delta T$ (D) $\frac{1}{9}(\alpha_1 - \alpha_2)\Delta T$

Q.13 The diagram shows an incomplete sketch of a PO box. Battery is connected between________. The unknown resistance is connected between ________. The galvanometer is connected between_________. The key K_2 is connected between __________.
Options:
(A) CD (B) DA (C) CE (D) DF
(E) DE (F) BF (G) CF

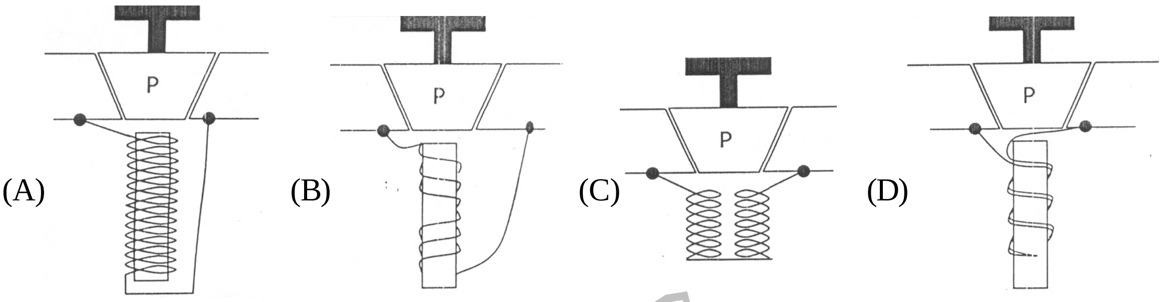


Q.14 For a post office Box, the graph of galvanometer deflection versus R (resistance pulled out of RB) for the ratio 100 : 1 is given as shown. A careless student pulls out two non consecutive values R as shown in the figure. Find the value of unknown resistance.
(A) 3.2 ohm (B) 3.24 ohm
(C) 3.206 ohm (D) None



Q.15 When we operate a wheat stone bridge then in starting the key of the battery is closed first and the key of the G is closed later. When the circuit is to be closed then switches are released in the opposite order. Why?
(A) Look at the diagram of the PO box, the switch is battery is always on the right hand hence it is easier to press it first.
(B) This is done to avoid the damage of the galvanometer due to induced emf.
(C) If the G switch is pressed before the battery switch then large sparking takes place at the battery switch.
(D) While disconnecting if we open the battery switch before the G switch then we can observe induced current in the circuit till the G switch is not opened.

Q.16 Identify which of the following diagrams represent the internal construction of the coils wound in a resistance box or PQ box?



Q.17 Which of the following reading is most accurate
(A) 4.00 cm (B) 0.004 mm (C) 40.00 cm (D) 4.00 m

Q.18 The least count of a stop watch is 1/5 sec. The time of 20 oscillations of a pendulum is measured to be 25 sec. The minimum percentage error in the measurement of time will be
(A) 0.1% (B) 0.8% (C) 1.8% (D) 8%

Q.19 A vernier callipers having 1 main scale division = 0.1 cm is designed to have a least count of 0.02 cm. If n be the number of divisions on vernier scale and m be the length of vernier scale, then
(A) n=10, m=0.5 cm (B) n=9, m=0.4 cm (C) n=10, m=0.8 cm (D) n=10 , m=0.2 cm

Q.20 Solve with due regard to significant digits

(i) $\sqrt{6.5-6.32}$ (ii) $\dfrac{2.91\times 0.3842}{0.080}$

Q.21 A body travels uniformly a distance of (13.8 ± 0.2)m in time (4.0 ± 0.3) sec. Calculate its velocity.

Q.22 The main scale of a vernier calipers reads in millimeter and its vernier is divided into 10 divisions which coincide with 9 divisions of the main scale. When the two jaws of the instrument touch each other the seventh division of the vernier scale coincide with a scale division and the zero of the vernier lies to the right of the zero of main scale. Furthermore, when a cylinder is tightly placed along its length between the two jaws, the zero of the vernier scale lies slightly to the left of 3.2 cm and the fourth vernier division coincides with a scale division. Calculate the measured length of the cylinder.

Q.23 A short circuit occurs in a telephone cable having a resistance of $0.45\ \Omega m^{-1}$. The circuit is tested with a Wheatstone bridge. The two resistors in the ratio arms of the Wheatstone bridge network have values of 100Ω and 1110Ω respectively. A balance condition is found when the variable resistor has a value of 400Ω. Calculate the distance down the cable, where the short has occurred.

Q.24 5.74 gm of a substance occupies a volume of 1.2 cm^3. Calculate its density with due regard for significant figures.

Q.25 The time period of oscillation of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

The length of the pendulum is measured as $l = 10 \pm 0.1$cm and the time period as $T = 0.5 \pm 0.02$ s. Determine percentage error in the value of g.

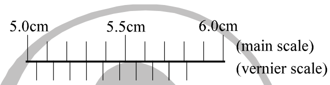
Q.26 A physical quantity P is related to four observables A, B, C and D as follows.

$$P = 4\pi^2 \frac{A^3 B^2}{\sqrt{C}\ D}$$

The percentage error of the measurement in A, B, C and D are 1%, 3% and 2%, 4% respectively. Determine the percentage error & absolute error in the quantity P. Value of P is calculated 3.763. Round off the result in scientific way.

Q.27 A glass prism of angle $A = 60^0$ gives minimum angle of deviation $\theta = 30^0$ with the max. error of 1^0 when a beam of parallel light passed through the prism during an experiment.
(i) Find the permissible error in the measurement of refractive index μ of the material of the prism.
(ii) Find the range of experimental value of refractive index 'μ'.

Q.28 In the given vernier calliper scale, the length of 1 main scale division is 1mm whereas the length of the vernier scale is 7.65 mm. Find the reading on the scale correct to significant digits as shown in the diagram.



ANSWER

Q.1 A   Q.2 B   Q.3 B   Q.4 D   Q.5 D   Q.6 (i) x = 0.6 msd, (ii) 6, 1st

Q.7 $L.C = l\left[\frac{1-\cos\theta}{\cos\theta}\right]$

Q.8 4 msd + 0.1 msd + 0.12 msd = 4.22 msd ; 4 msd + 0.1 msd + 1.12 msd = 5.22 msd & so on

Q.9 A   Q.10 CD, AB, C   Q.11 B   Q.12 A   Q.13 CE, CD, DF, BF

Q.14 B   Q.15 B,C,D   Q.16 D   Q.17 C   Q.18 B

Q.19 C   Q.20 (i) 0.4 ; (ii) 14   Q.21 $v = (3.5 \pm 0.31)$ m/s

Q.22 3.07 cm   Q.23 40 m   Q.24 4.8 g/cm^3 Q.25 5%

Q.26 14%, 0.53, 3.76   Q.27 5π/18% , $\sqrt{2}\left[1+\frac{\pi}{360}\right] > \mu > \sqrt{2}\left[1-\frac{\pi}{360}\right]$

Q.28 5.045 cm

